

# NUCLEAR CROSS SECTIONS FOR HEAVY CHARGED-PARTICLE TRANSPORT

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The need to develop suitable methods for describing the interactions and transport of high-energy, heavy charged-particles through extended matter is important for a variety of applications including astronaut exposure to space radiations, spacecraft shielding, radiobiological studies, accelerator shield design, and clinical uses in cancer therapy. For most of these applications, the transport equation can be written, neglecting target fragmentation and using the straight-ahead approximation, as

$$\left[ \frac{\partial}{\partial x} - \frac{\partial}{\partial E} \tilde{S}_j(E) + \rho \sigma_j(E) \right] \phi_j(x, E) = \sum_{k > j} m_{jk}(E) \rho \sigma_k(AE) \phi_k(x, E) \quad (1)$$

where

$\phi_j(x, E)$  - flux of  $j$ th ions

$A_j$  - atomic mass number

$E$  - energy (MeV/Nucleon)

$\sigma_j(E)$  - nuclear absorption cross section

$\tilde{S}_j(E)$  - stopping power

$m_{jk}(E)$  - multiplicity of ion  $j$  produced by ion  $k$

$\rho$  - target number density.

Crucial to satisfactorily solving equation (1) are accurate values for the absorption and fragmentation cross sections. Unfortunately, experimental data for these cross sections are sparse. Hence, the theoretical and semi-empirical methods must be utilized to provide the necessary cross section values.

## Nuclear Absorption Cross Sections

Because solutions to the transport equation (eq. (1)) possess an exponential dependence upon the input absorption cross sections, the need for accurate values is paramount since even relatively small errors will result in rather large transmitted flux uncertainties. Absorption cross sections are typically estimated using quantum-mechanical multiple-scattering models<sup>1</sup>, or from simple parameterizations based upon classical collision models. The main advantage of the quantum mechanical models is the accuracy of their predictions. Their major disadvantages are the non-trivial nature of the calculations and the extensive memory requirements if the cross sections are stored in tables. In one theoretical model<sup>1</sup>, the absorption cross sections are obtained from

$$\sigma_{\text{abs}} = \int d^2\vec{b} \{1 - \exp[-2 \operatorname{Im} \chi(\vec{b})]\} \quad (2)$$

where the complex phase function,  $\chi$ , written in terms of the nucleus-nucleus optical potential,  $V(\vec{b}, z)$ , is

$$\chi(\vec{b}) = -[mA_P A_T / k(A_P + A_T)] \int_{-\infty}^{\infty} V(\vec{b}, z) dz \quad (3)$$

with

$$V(\vec{b}, z) = A_P A_T \int d^3\vec{\xi}_T \rho_T(\vec{\xi}_T) \int d^3\vec{y} \rho_P(\vec{b} + \vec{z} + \vec{\xi}_T + \vec{y}) \tilde{t}(\vec{e}, \vec{y}) . \quad (4)$$

Symbols in equations (2)-(4) are

$m$  - nucleon mass

$\vec{b}$  - impact parameter

$A_i$  ( $i = P, T$ ) - projectile (P) and target (T) mass numbers

$k$  - projectile momentum

$\bar{t}$  - constituent-averaged, energy-dependent two-nucleon transition amplitude

$\rho_i$  ( $i = P, T$ ) - nuclear densities of colliding nuclei .

Detailed calculations, extensive tables of predictions, and comparisons with experimental data are presented elsewhere.<sup>3</sup>

As an alternative, parameterizations based upon classical collision models are useful because of their simplicity. The most commonly used ones are of the Bradt-Peters form<sup>2</sup>

$$\sigma_{\text{abs}} = \pi r_0^2 (A_P^{1/3} + A_T^{1/3} - \delta)^2 \quad (5)$$

where  $r_0$  and  $\delta$  are energy-independent parameters adjusted to fit available experimental data. These parameterizations are reasonably accurate (within 5 percent) at energies above  $\approx 1$  GeV/Nucleon where the cross sections are nearly asymptotic. For lower energies substantial differences exist due to the cross section energy dependence. Recently, fully energy-dependent parameterizations of proton-nucleus<sup>4</sup> and nucleus-nucleus<sup>5</sup> absorption cross sections have been developed for use in transport studies. These energy-dependent parameterizations typically agree to within 10 percent of the more detailed optical model potential results obtained with equation (2).

## Fragmentation Cross Sections

Unlike absorptive processes, the physics underlying fragmentation processes is not understood and remains a subject of current interest. Much of the sparse experimental data base is relevant only to studies of the underlying physics and not directly useful in transport studies. Semi-empirical formulae have been developed to predict fragmentation cross sections<sup>6</sup> but typical inaccuracies are ~30 percent. Current theoretical efforts center upon a formalism called an Abrasion-Ablation Model. In abrasion, portions of the nuclear volume are sheared off in the collision. The remaining nucleus, highly misshapen and excited, then evaporates energy and particles (ablation step) until a final fragment is formed in its ground state. The abrasion cross section is calculated using geometric or quantum mechanical formalisms and the ablation probabilities estimated using existing evaporation codes.

## Cross Section Results

Representative results for carbon-carbon collisions are listed in the table. Presented are absorption cross sections at several energies and fragmentation cross sections at 2.1 GeV/Nucleon. The cross sections include experimental results<sup>8,9,10</sup>, and predictions from parameterizations and quantum mechanical models. For absorption cross sections, the energy independent parameterization fails at low energies,

whereas the optical model calculations and energy dependent parameterization results are in good agreement with the experimental data. For the fragmentation, reasonable agreement is obtained for some isotopes, however, large disagreements exist for others. Clearly much physics remains to be understood about the fragmentation process.

## References

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Table 1.- Representative Carbon-Carbon Nuclear Cross Sections

Absorption Cross Section, mb				
Energy MeV/Nucleon	Optical Model	Parameterization		Experiment (ref. 8,9)
		Energy Dependent	Energy Independent	
30	1174	1117	873	1315 $\pm$ 40
83	920	927	873	960 $\pm$ 30
870	853	901	873	939 $\pm$ 49
2100	862	895	873	888 $\pm$ 50
2.1 GeV/Nucleon Fragmentation Cross Sections, mb				
Isotope produced	Abrasion-Ablation Model		Experiment (ref. 10)	
$^{11}\text{C}$	55.9		4.65 $\pm$ 2.3	
$^{10}\text{C}$	13.7		4.11 $\pm$ .22	
$^{11}\text{B}$	55.9		53.8 $\pm$ 2.7	
$^{10}\text{B}$	31.0		35.1 $\pm$ 3.4	
$^9\text{B}$	61.8		-	
$^8\text{B}$	13.4		1.72 $\pm$ .13	
$^{10}\text{Be}$	14.0		5.81 $\pm$ .29	
$^9\text{Be}$	59.9		10.6 $\pm$ .5	
$^7\text{Be}$	54.1		18.6 $\pm$ .9	
$^7\text{Li}$	55.9		21.5 $\pm$ 1.1	
$^6\text{Li}$	86.1		30.0 $\pm$ 2.4	